The Assignment Problem – AJ’s Method

Suppose there are jobs and persons to fill these jobs. People fit some jobs better than other jobs. Thus, each person has a rating for each jobs. In other words, Person A has a rating to fill Job 1, and a rating to fill Job 2. Person B has a rating to fill Job 1, and a rating to fill Job 2. If there are three people and three jobs, there would be nine ratings.

The *lower* the rating, the better it is. These ratings are **relative** – they provide an ordering of preference. The numerical values themselves indicate nothing, and a rating of “2” is *not* twice as bad as a rating of “1.”

Our goal: assign each person to exactly one job in such a way as to minimize the total of ratings.

The x **ratings matrix**  is where is the rating of the person for the job .

The x **assignment matrix**  is such that

The assignment problem is

The dual of Problem (B) is

It follows from the complementary slackness theorem (as applied to problems of the forms *A* and *B*) that is optimal for *A* if and only if there exist and such that , , satisfy

1. is feasible for .
2. , are feasible for . That is,

I will present an alternate to the Hungarian Method to solve this problem.

**AJ’s Method -** Given C, an x matrix:

1. Row by row, from left to right, circle the minimum value. There should only be one circled value for each row. In case of a tie, choose and circle the left most value.
2. A) If all columns have a circled value, we have an optimal solution.

B) If there are columns without a circled value, label those columns and a “u.” These are **unsaturated** columns.

1. Using the unsaturated columns, make a new table that consists of only the unsaturated columns. The new table will be written on the right end of our initial table. Write the new table as if the saturated columns did not exist.
2. In our original table, find all columns that have only one assignment (circled value). This signifies an assignment that is optimal. With this, we may cross out the corresponding row of our new table. We may also cross out the corresponding column.